This analysis is of a single stage expansion engine compared to a 3 stage expansion steam engine.

Include the Rankine compression cycle functions by referance below.

Reference:C:\Program Files\MathSoft\Mathcad 2001 Professional\steam\cycle\Rankin.mcd

High compression single stage engine expansing to near atmospheric pressure.

In := ST_ptdata(1000, 950, 1, 1)   xh := 14.66   cy := Rankine_p(In, 17, xh, 3%, 1000)

		Inlet	Steam Cutoff	Cycle Expansion	Points Exhaust	Compression	First Colume	Steam Properties
cy =	1	1000	1000	17	14.66	1000	N/A	Pressure
	0.03	950	1058.153	219.436	211.876	1099.844	Clearance	Temperature
	0.01	0.795	0.869	22.235	25.623	0.897	Cutoff	Specific Volume
	0.827	1329.912	1377.535	1035.765	1036.203	1395.928	Exhaust	Internal Energy
	0	1477.073	1538.298	1105.715	1105.715	1561.825	N/A	Enthalpy
	0.722	1.633	1.675	1.675	1.69	1.69	residual	Entropy
	0.278	1	1	0.951	0.954	1	inlet	Quality

$\mathrm{CycleWork(cy)} = 406 \qquad \mathrm{CycleEff(cy)} = 30.5512\% \qquad \frac{cy_{\boldsymbol{v},\boldsymbol{ex}}}{cy_{\boldsymbol{v},\boldsymbol{cu}}} = 25.59$ expansion ratio

In order to get this to work the above single cycle uses a clearance of 3%. That makes the cutoff be 1.021% to expand down to 17 PSIA. Geting decent efficiency of 30.5%. I figure that 3% is possable the smallest clearance one might be able to get. But that leaves the cutoff at an impratictly low value. There is no room here to do cutoff clearance controll and maintain the end of expansion pressure at 17 PSIA. I doubt an engine could be built that would run with this low clearance or cutoff. It produces 406 BTU of output work per lb of inlet steam. It manages to use 27.7% inlet steam to 72.3% residual recucled steam.

Note the cycle metrix above. To it's right is the content discriptions. The first colume is cycle parameters. The other five columes are steam state points. The state point columes are labeled above. They are inlet, cutoff,end of expansion, exhaust, and compression. These are calculated using a constant enthalpy pressure drop to exhaust pressure (End of expansion and exhaust enthalpy being equal).

The following is three cycle calculations figuring for an equlivant 3 stage compound engine. The work of each stage has been balanced somewhat by adjusting interstage parameters. Here I am trying to get an idea of the relative displacements required to produce the same power. It is said that a compound must be larger then a single stage. But that dosn't prove out here. It's hard to decide on equilivant engines here. I tried to get the isentropic (work producing) expansion to be the same. Not an easy task sense the compound expansions equal to the single stage engine while balancing stage outputs.

$cl := 5\%$ $xh_{hp} := 466$ $xh_{mp} := 105.65$ $ex_{hp} := xh_{hp} + 14$ $ex_{mp} := xh_{mp} + 14$ $ex_{lp} := xh + 14$

First stage cycle calculation.

$In_{hp} := \mathrm{ST\_ptdata}(1480, 800, 1, 1)$ $cy_{hp} := \mathrm{Rankine\_p}\left(In_{hp}, ex_{hp}, xh_{hp}, cl, 1480\right)$

$$cy_{hp} = \begin{pmatrix} 1 & 1480 & 1480 & 480 & 466 & 1480 \\ 0.05 & 800 & 800.685 & 514.337 & 511.769 & 806.02 \\ 0.385 & 0.442 & 0.442 & 1.067 & 1.1 & 0.445 \\ 0.073 & 1244.054 & 1244.415 & 1149.789 & 1149.767 & 1247.213 \\ 0 & 1365.059 & 1365.529 & 1244.603 & 1244.603 & 1369.173 \\ 0.114 & 1.509 & 1.51 & 1.51 & 1.513 & 1.513 \\ 0.886 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\mathrm{CycleWork}\left(cy_{hp}\right) = 123.6$

$\mathrm{CycleEff}\left(cy_{hp}\right) = 10.2\,\%$

$$\frac{{cy_{hp}}_{\boldsymbol{v},\boldsymbol{ex}}}{{cy_{hp}}_{\boldsymbol{v},\boldsymbol{cu}}} = 2.4138$$

Second stage cycle calculation

$In_{mp} := \mathrm{ST\_ptdata}\left({cy_{hp}}_{\boldsymbol{p},\boldsymbol{xh}}, {cy_{hp}}_{\boldsymbol{h},\boldsymbol{xh}}, \boldsymbol{h}, 1\right)$ $cy_{mp} := \mathrm{Rankine\_p}\left(In_{mp}, ex_{mp}, xh_{mp}, cl, {In_{mp}}_{\boldsymbol{p}}\right)$

$$cy_{mp} = \begin{pmatrix} 1 & 466 & 466 & 119.65 & 105.65 & 466 \\ 0.05 & 511.769 & 514.634 & 341.046 & 331.822 & 531.76 \\ 0.283 & 1.1 & 1.105 & 3.489 & 3.939 & 1.136 \\ 0.123 & 1149.767 & 1151.338 & 1054.365 & 1054.612 & 1160.506 \\ 0 & 1244.603 & 1246.633 & 1131.619 & 1131.619 & 1258.482 \\ 0.146 & 1.513 & 1.515 & 1.515 & 1.527 & 1.527 \\ 0.854 & 1 & 1 & 0.933 & 0.936 & 1 \end{pmatrix}$$

$\mathrm{CycleWork}\left(cy_{mp}\right) = 123.6$

$\mathrm{CycleEff}\left(cy_{mp}\right) = 11.3\,\%$

$$\frac{{cy_{mp}}_{\boldsymbol{v},\boldsymbol{ex}}}{{cy_{mp}}_{\boldsymbol{v},\boldsymbol{cu}}} = 3.1574$$

Third stage cycle calculation

$In_{lp} := \mathrm{ST\_ptdata}\left({cy_{mp}}_{\boldsymbol{p},\boldsymbol{xh}}, {cy_{mp}}_{\boldsymbol{h},\boldsymbol{xh}}, \boldsymbol{h}, 1\right)$ $cy_{lp} := \mathrm{Rankine\_p}\left(In_{lp}, ex_{lp}, xh, cl, {In_{lp}}_{\boldsymbol{p}}\right)$

$$cy_{lp} = \begin{pmatrix} 1 & 105.65 & 105.65 & 28.66 & 14.66 & 105.65 \\ 0.05 & 331.822 & 331.822 & 247.731 & 211.876 & 336.161 \\ 0.282 & 3.939 & 3.978 & 12.57 & 23.971 & 4.236 \\ 0.233 & 1054.612 & 1062.186 & 979.318 & 980.954 & 1107.865 \\ 0 & 1131.619 & 1139.965 & 1045.984 & 1045.984 & 1190.687 \\ 0.141 & 1.527 & 1.537 & 1.537 & 1.601 & 1.601 \\ 0.859 & 0.936 & 0.946 & 0.876 & 0.892 & 1 \end{pmatrix}$$

$\mathrm{CycleWork}\left(cy_{lp}\right) = 123.6$

$\mathrm{CycleEff}\left(cy_{lp}\right) = 12.6\,\%$

$$\frac{{cy_{lp}}_{\boldsymbol{v},\boldsymbol{ex}}}{{cy_{lp}}_{\boldsymbol{v},\boldsymbol{cu}}} = 3.1596$$

The single stage engines expansion ratio:

$$\frac{cy_{\boldsymbol{v},\boldsymbol{ex}}}{cy_{\boldsymbol{v},\boldsymbol{cu}}} = 25.5946$$

Trying to get the engines expansion equal. Hard to do. Close enough. Could make MathCad do the balancing but would take time to develope the programming. The efficiencies are close enough. So going with this.

The compound combined stages expansion ratio:

$$\frac{cyhp_{\boldsymbol{v},\boldsymbol{ex}}}{cyhp_{\boldsymbol{v},\boldsymbol{cu}}} \cdot \frac{cymp_{\boldsymbol{v},\boldsymbol{ex}}}{cymp_{\boldsymbol{v},\boldsymbol{cu}}} \cdot \frac{cylp_{\boldsymbol{v},\boldsymbol{ex}}}{cylp_{\boldsymbol{v},\boldsymbol{cu}}} = 24.08$$

Expansion doing work.

Compound total over all expansion ratio HP(first stage) cutoff point speciic volume to LP (last stage) end of expansion specific volume.

$$\frac{cylp_{\boldsymbol{v},\boldsymbol{ex}}}{cyhp_{\boldsymbol{v},\boldsymbol{cu}}} = 28.4243$$

Includes non work and work expansion.

The following figures second and third stage scaling parameters so as to use an equal amount steam.

$$S_{lp} := \frac{cyhp_{\boldsymbol{im},\boldsymbol{PA}}}{cylp_{\boldsymbol{im},\boldsymbol{PA}}} \qquad S_{lp} = 1.0316 \qquad S_{mp} := \frac{cyhp_{\boldsymbol{im},\boldsymbol{PA}}}{cymp_{\boldsymbol{im},\boldsymbol{PA}}} \qquad S_{mp} = 1.0376$$

The following is used to balance stage output.

$$\frac{\mathrm{CycleWO}(cy_{hp}) + S_{mp}\cdot\mathrm{CycleWO}(cy_{mp}) + S_{lp}\cdot\mathrm{CycleWO}(cy_{lp})}{3} = 109.4649$$

Referance used for stage output. Each stages scaled output shouled equal this

Scaled output work of each stage in BTU:

Varied inter stage pressure to get these close.

$$\mathrm{CycleWO}(cy_{hp}) = 109.4722 \qquad S_{mp}\cdot\mathrm{CycleWO}(cy_{mp}) = 109.4671 \qquad S_{lp}\cdot\mathrm{CycleWO}(cy_{lp}) = 109.4553$$

Some values of interest

$$\mathrm{CycleWork}(cy_{hp}) + \mathrm{CycleWork}(cy_{mp}) + \mathrm{CycleWork}(cy_{lp}) = 370.7087$$

BTU/lb of steam

$$\frac{\mathrm{CycleWO}(cy_{hp}) + S_{mp}\cdot\mathrm{CycleWO}(cy_{mp}) + S_{lp}\cdot\mathrm{CycleWO}(cy_{lp})}{\left(cyhp_{\boldsymbol{h},\boldsymbol{in}} - \mathrm{ST\_ptdata}(14.696, 180, 1, 1)_{\boldsymbol{h}}\right) cyhp_{\boldsymbol{im},\boldsymbol{PA}}} = 30.4598\,\%$$

Engine efficiency

Down to figuring relative engine size for equal amounts of output work.

To do that I figure the output work in ftlb per cubic in of diplacement.

$$\mathrm{Work}_{1S} := \frac{\mathrm{CycleWO(cy)} \cdot \boldsymbol{J}}{\mathrm{cy}_{\boldsymbol{v},\,\boldsymbol{ex}} \cdot 12 \cdot 12 \cdot 12}$$

The output work for the single stage engine per cubic inch.

$$\mathrm{Work}_{3S} := \frac{\left(\mathrm{CycleWO}\left(\mathrm{cy}_{hp}\right) + S_{mp} \cdot \mathrm{CycleWO}\left(\mathrm{cy}_{mp}\right) + S_{lp} \cdot \mathrm{CycleWO}\left(\mathrm{cy}_{lp}\right)\right) \cdot \boldsymbol{J}}{\left(\mathrm{cy}_{hp_{\boldsymbol{v},\,\boldsymbol{ex}}} + S_{mp} \cdot \mathrm{cy}_{mp_{\boldsymbol{v},\,\boldsymbol{ex}}} + S_{lp} \cdot \mathrm{cy}_{lp_{\boldsymbol{v},\,\boldsymbol{ex}}}\right) \cdot 12 \cdot 12 \cdot 12}$$

Same for three stage engne.

CycleWO figures the output work of the cycle for each pound of steam in the cycle. Here the pound of steam in the cycle includes both inlet and residual parts. The cycle function figure a cycle based in a pound of steam in the engine cycle. The total displacement is figured as stage displacements containing a pound of steam with the second and last stage scaled to use the same amount of inlet as the first stage. The displacement is based on the specific volume at the end of expansion point.

The output work is converted into foot pounds.

And the results are here:

$\mathrm{Work}_{1S} = 2.2828$ ft lb/CuFt $\mathrm{Work}_{3S} = 8.3763$ ft lb/CuFt

Comparing the work output per cubic inch of displacement we find the compound produces 3.67 times the work of a single stage for a given displacement.

$$\frac{\mathrm{Work}_{3S}}{\mathrm{Work}_{1S}} = 3.6693$$

Or for equal output the single stage must have 3.67 times the displacement of the 3 stage compound.

$$\frac{\mathrm{cy}_{hp_{\boldsymbol{im},\,\boldsymbol{PA}}}}{\mathrm{cy}_{\boldsymbol{im},\,\boldsymbol{PA}}} = 3.1911$$

Right here is the reasion the compound is so much more space saving. We are running a compression cycle and when you compare the inlet part of the single expansion to the first stage of the compounds we find 3.19 time the inlet steam going through the engine. It makes sense that the compound would be a lot smaller.

$$\frac{\mathrm{cy}_{mp_{\boldsymbol{im},\,\boldsymbol{PA}}} \cdot S_{mp}}{\mathrm{cy}_{\boldsymbol{im},\,\boldsymbol{PA}}} = 3.1911$$

$$\frac{\mathrm{cy}_{lp_{\boldsymbol{im},\,\boldsymbol{PA}}} \cdot S_{lp}}{\mathrm{cy}_{\boldsymbol{im},\,\boldsymbol{PA}}} = 3.1911$$

HP/cuin @3600RPM

$$\mathrm{Work}_{1S} \cdot \frac{3600}{60 \cdot 550} = 0.249$$

$$\mathrm{Work}_{3S} \cdot \frac{3600}{60 \cdot 550} = 0.9138$$