

This analysis is of a single stage expansion engine compared to a 3 stage expansion steam engine.

Include the Rankine compression cycle functions by reference below. $ST_limit(7) = 7$

☞ Reference: C:\Program Files\MathSoft\Mathcad 2001 Professional\steam\cycle\Rankin.mcd

High compression single stage engine expanding to near atmospheric pressure.

$$P := 3200 \quad T := 1200$$

$$In := ST_pdata(P, T, 1, 1) \quad xh_S := 220 \quad cy := Rankine_c(In, 6\%, xh_S, 8\%, P)$$

	Inlet	Steam Cutoff	Cycle Expansion	Points Exhaust	Points Compression	First Colume	Steam Properties
cy =	7.714	3200	3200	231.781	220	3200	N/A Pressure
	0.08	1200	1218.061	462.576	459.894	1232.045	Clearance Temperature
	0.06	0.283	0.287	2.214	2.333	0.29	Cutoff Specific Volume
	0.563	1402.905	1412.069	1148.708	1148.693	1419.145	Exhaust Internal Energy
	0	1570.335	1582.09	1243.677	1243.677	1591.154	N/A Enthalpy
	0.565	1.575	1.582	1.582	1.587	1.587	residual Entropy
	0.435	1	1	1	1	1	inlet Quality

$$CycleWork(cy) = 337.7 \quad CycleEff(cy) = 23.7\% \quad \frac{cy_{v,ex}}{cy_{v,cu}} = 7.71 \quad \text{expansion ratio}$$

In order to get this to work the above single cycle uses a clearance of 3%. That makes the cutoff be 1.021% to expand down to 17 PSIA. Getting decent efficiency of 30.5%. I figure that 3% is possible the smallest clearance one might be able to get. But that leaves the cutoff at an impracticly low value. There is no room here to do cutoff clearance controll and maintain the end of expansion pressure at 17 PSIA. I doubt an engine could be built that would run with this low clearance or cutoff. It produces 406 BTU of output work per lb of inlet steam. It manages to use 27.7% inlet steam to 72.3% residual recucled steam.

Note the cycle metrix above. To it's right is the content discriptions. The first colume is cycle parameters. The other five columes are steam state points. The state point columes are labeled above. They are inlet, cutoff,end of expansion, exhaust, and compression. These are calculated using a constant enthalpy pressure drop to exhaust pressure (End of expansion and exhaust enthalpy being equal).

The following is three cycle calculations figuring for an equivant 3 stage compound engine. The work of each stage has been balanced somewhat by adjusting interstage parameters. Here I am trying to get an idea of the relative displacements required to produce the same power. It is said that a compound must be larger then a single stage. But that doesn't prove out here. It's hard to decide on equivant engines here. I tried to get the isentropic (work producing) expansion to be the same. Not an easy task sense the compound expansions equal to the single stage engine while balancing stage outputs.

$$cl := 5\% \quad x_{hp} := 2120 \quad x_{mp} := 550 \quad x_{lp} := 100 \quad ex_{hp} := x_{hp} + 10 \quad ex_{mp} := x_{mp} + 10 \quad ex_{lp} := x_{lp} + 10$$

First stage cycle calculation. $P := 6000$ $T := 1150$

$$In_{hp} := ST_pdata(P, T, 1, 1) \quad cy_{hp} := Rankine_p(In_{hp}, ex_{hp}, x_{hp}, cl, P)$$

$$cy_{hp} = \begin{pmatrix} 1 & 6000 & 6000 & 2130 & 2120 & 6000 \\ 0.05 & 1150 & 1150.095 & 806.505 & 805.768 & 1150.908 \\ 0.429 & 0.131 & 0.131 & 0.287 & 0.288 & 0.131 \\ 0.06 & 1320.028 & 1320.089 & 1219.785 & 1219.78 & 1320.613 \\ 0 & 1465.36 & 1465.438 & 1332.824 & 1332.824 & 1466.115 \\ 0.104 & 1.45 & 1.45 & 1.45 & 1.451 & 1.451 \\ 0.896 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} \text{CycleWork}(cy_{hp}) &= 133.1 \\ \text{CycleEff}(cy_{hp}) &= 10.1\% \\ \frac{cy_{hp,v,ex}}{cy_{hp,v,cu}} &= 2.1907 \end{aligned}$$

Second stage cycle calculation

$$In_{mp} := ST_pdata(cy_{hp,p,xh}, cy_{hp,h,xh}, h, 1) \quad cy_{mp} := Rankine_p(In_{mp}, ex_{mp}, x_{mp}, cl, In_{mp,p})$$

$$cy_{mp} = \begin{pmatrix} 1 & 2120 & 2120 & 560 & 550 & 2120 \\ 0.05 & 805.768 & 806.166 & 478.843 & 476.94 & 808.715 \\ 0.317 & 0.288 & 0.288 & 0.824 & 0.839 & 0.29 \\ 0.095 & 1219.78 & 1220.03 & 1116.752 & 1116.726 & 1221.626 \\ 0 & 1332.824 & 1333.15 & 1202.161 & 1202.161 & 1335.23 \\ 0.136 & 1.451 & 1.451 & 1.451 & 1.452 & 1.452 \\ 0.864 & 1 & 1 & 0.997 & 0.997 & 1 \end{pmatrix}$$

$$\begin{aligned} \text{CycleWork}(cy_{mp}) &= 132.4 \\ \text{CycleEff}(cy_{mp}) &= 11.2\% \\ \frac{cy_{mp,v,ex}}{cy_{mp,v,cu}} &= 2.8583 \end{aligned}$$

Third stage cycle calculation

$$In_{lp} := ST_pdata(cy_{mp,p,xh}, cy_{mp,h,xh}, h, 1) \quad cy_{lp} := Rankine_p(In_{lp}, ex_{lp}, x_{lp}, cl, In_{lp,p})$$

$$cy_{lp} = \begin{pmatrix} 1 & 550 & 550 & 110 & 100 & 550 \\ 0.05 & 476.94 & 476.94 & 334.787 & 327.823 & 486.197 \\ 0.2 & 0.839 & 0.842 & 3.539 & 3.885 & 0.859 \\ 0.176 & 1116.726 & 1118.493 & 1005.25 & 1005.394 & 1124.849 \\ 0 & 1202.161 & 1204.152 & 1077.292 & 1077.292 & 1212.314 \\ 0.196 & 1.452 & 1.455 & 1.455 & 1.463 & 1.463 \\ 0.804 & 0.997 & 1 & 0.874 & 0.876 & 1 \end{pmatrix}$$

$$\begin{aligned} \text{CycleWork}(cy_{lp}) &= 133 \\ \text{CycleEff}(cy_{lp}) &= 12.6\% \\ \frac{cy_{lp,v,ex}}{cy_{lp,v,cu}} &= 4.2052 \end{aligned}$$

The single stage engines expansion ratio:

$$\frac{cy_{v,ex}}{cy_{v,cu}} = 7.7118$$

Trying to get the engines expansion equal. Hard to do. Close enough. Could make MathCad do the balancing but would take time to develop the programming. The efficiencies are close enough. So going with this.

The compound combined stages expansion ratio:

$$\frac{cy_{hp,v,ex} \cdot cy_{mp,v,ex} \cdot cy_{lp,v,ex}}{cy_{hp,v,cu} \cdot cy_{mp,v,cu} \cdot cy_{lp,v,cu}} = 26.3318 \quad \text{Expansion doing work.}$$

Compound total over all expansion ratio HP(first stage) cutoff point specific volume to LP (last stage) end of expansion specific volume.

$$\frac{cy_{lp,v,ex}}{cy_{hp,v,cu}} = 27.0353 \quad \text{Includes non work and work expansion.}$$

The following figures second and third stage scaling parameters so as to use an equal amount steam.

$$S_{mp} := \frac{cy_{hp,im,PA}}{cy_{mp,im,PA}} \quad S_{mp} = 1.0362 \quad S_{lp} := \frac{cy_{hp,im,PA}}{cy_{lp,im,PA}} \quad S_{lp} = 1.1143$$

The following is used to balance stage output.

$$\frac{\text{CycleWO}(cy_{hp}) + S_{mp} \cdot \text{CycleWO}(cy_{mp}) + S_{lp} \cdot \text{CycleWO}(cy_{lp})}{3} = 119.012$$

Reference used for stage output. Each stages scaled output should equal this

Scaled output work of each stage in BTU:

Varied inter stage pressure to get these close.

$$\text{CycleWO}(cy_{hp}) = 119.2548 \quad S_{mp} \cdot \text{CycleWO}(cy_{mp}) = 118.6273 \quad S_{lp} \cdot \text{CycleWO}(cy_{lp}) = 119.1539$$

Some values of interest

$$\text{CycleWork}(cy_{hp}) + S_{mp} \cdot \text{CycleWork}(cy_{mp}) + S_{lp} \cdot \text{CycleWork}(cy_{lp}) = 418.5756 \quad \text{BTU/lb of steam}$$

$$\frac{\text{CycleWO}(cy_{hp}) + S_{mp} \cdot \text{CycleWO}(cy_{mp}) + S_{lp} \cdot \text{CycleWO}(cy_{lp})}{(cy_{hp,h,in} - ST_ptdata(14.696, 180, 1, 1)_h) cy_{hp,im,PA}} = 30.2557\% \quad \text{Engine efficiency}$$

Down to figuring relative engine size for equal amounts of output work.

To do that I figure the output work in ftlb per cubic in of displacement.

$$\text{Work}_{1S} := \frac{\text{CycleWO}(\text{cy}) \cdot \mathbf{J}}{\text{cy}_{\mathbf{v, ex}} \cdot 12 \cdot 12 \cdot 12} \quad \text{The output work for the single stage engine per cubic inch.}$$

$$\text{Work}_{3S} := \frac{(\text{CycleWO}(\text{cy}_{\text{hp}}) + S_{\text{mp}} \cdot \text{CycleWO}(\text{cy}_{\text{mp}}) + S_{\text{lp}} \cdot \text{CycleWO}(\text{cy}_{\text{lp}})) \cdot \mathbf{J}}{(\text{cy}_{\text{hp}, \mathbf{v, ex}} + S_{\text{mp}} \cdot \text{cy}_{\text{mp}, \mathbf{v, ex}} + S_{\text{lp}} \cdot \text{cy}_{\text{lp}, \mathbf{v, ex}}) \cdot 12 \cdot 12 \cdot 12} \quad \text{Same for three stage engine.}$$

CycleWO figures the output work of the cycle for each pound of steam in the cycle. Here the pound of steam in the cycle includes both inlet and residual parts. The cycle function figure a cycle based in a pound of steam in the engine cycle. The total displacement is figured as stage displacements containing a pound of steam with the second and last stage scaled to use the same amount of inlet as the first stage. The displacement is based on the specific volume at the end of expansion point.

The output work is converted into foot pounds.

And the results are here:

$$\text{Work}_{1S} = 29.8995 \quad \text{ft lb/in}^3 \quad \text{Work}_{3S} = 31.6215 \quad \text{ft lb/in}^3$$

Comparing the work output per cubic inch of displacement we find the compound produces 3.67 times the work of a single stage for a given displacement.

$$\frac{\text{Work}_{3S}}{\text{Work}_{1S}} = 1.0576 \quad \text{Or for equal output the single stage must have times the displacement of the 3 stage compound.}$$

$$\text{Work}_{1S} \cdot \frac{3600}{60 \cdot 550} = 3.2618 \quad \text{HP/in}^3 \text{ @3600RPM}$$

$$\text{Work}_{3S} \cdot \frac{3600}{60 \cdot 550} = 3.4496 \quad \text{HP/in}^3 \text{ @3600RPM}$$

A look at the efficiency ratio.

$$\frac{\text{CycleWO}(\text{cy}_{\text{hp}}) + S_{\text{mp}} \cdot \text{CycleWO}(\text{cy}_{\text{mp}}) + S_{\text{lp}} \cdot \text{CycleWO}(\text{cy}_{\text{lp}})}{\frac{(\text{cy}_{\text{hp}, \mathbf{h, in}} - \text{ST_ptdata}(14.696, 180, 1, 1)_{\mathbf{h}}) \text{cy}_{\text{lp}, \mathbf{im, PA}}}{\text{CycleEff}(\text{cy})}} = 1.4201$$