This analysis is of a single stage expansion engine compared to a 3 stage expansion steam engine.

Include the Rankine compression cycle functions by referance below. $ST\_limit(7) = 7$

➔ Reference:C:\Program Files\MathSoft\Mathcad 2001 Professional\steam\cycle\Rankin.mcd

High compression single stage engine expansing to near atmospheric pressure.

$P := 3200 \quad T := 1200$

$In := ST\_ptdata(P, T, 1, 1) \qquad xh_S := 220 \qquad cy := Rankine\_c(In, 6\%, xh_S, 8\%, P)$

	First Colume	Inlet	Steam Cycle Points: Cutoff	Expansion	Exhaust	Compression	First Colume	Steam Properties
cy =	7.714	3200	3200	231.781	220	3200	N/A	Pressure
	0.08	1200	1218.061	462.576	459.894	1232.045	Clearance	Temperature
	0.06	0.283	0.287	2.214	2.333	0.29	Cutoff	Specific Volume
	0.563	1402.905	1412.069	1148.708	1148.693	1419.145	Exhaust	Internal Energy
	0	1570.335	1582.09	1243.677	1243.677	1591.154	N/A	Enthalpy
	0.565	1.575	1.582	1.582	1.587	1.587	residual	Entropy
	0.435	1	1	1	1	1	inlet	Quality

$CycleWork(cy) = 337.7 \qquad CycleEff(cy) = 23.7\% \qquad \frac{cy_{\boldsymbol{v},\boldsymbol{ex}}}{cy_{\boldsymbol{v},\boldsymbol{cu}}} = 7.71$ expansion ratio

In order to get this to work the above single cycle uses a clearance of 3%. That makes the cutoff be 1.021% to expand down to 17 PSIA. Geting decent efficiency of 30.5%. I figure that 3% is possable the smallest clearance one might be able to get. But that leaves the cutoff at an impratictly low value. There is no room here to do cutoff clearance controll and maintain the end of expansion pressure at 17 PSIA. I doubt an engine could be built that would run with this low clearance or cutoff. It produces 406 BTU of output work per lb of inlet steam. It manages to use 27.7% inlet steam to 72.3% residual recucled steam.

Note the cycle metrix above. To it's right is the content discriptions. The first colume is cycle parameters. The other five columes are steam state points. The state point columes are labeled above. They are inlet, cutoff,end of expansion, exhaust, and compression. These are calculated using a constant enthalpy pressure drop to exhaust pressure (End of expansion and exhaust enthalpy being equal).

The following is three cycle calculations figuring for an equlivant 3 stage compound engine. The work of each stage has been balanced somewhat by adjusting interstage parameters. Here I am trying to get an idea of the relative displacements required to produce the same power. It is said that a compound must be larger then a single stage. But that dosn't prove out here. It's hard to decide on equilivant engines here. I tried to get the isentropic (work producing) expansion to be the same. Not an easy task sense the compound expansions equal to the single stage engine while balancing stage outputs.

$cl := 5\%$ $xh_{hp} := 1134$ $xh_{mp} := 197$ $xh_{lp} := 14.696$ $ex_{hp} := xh_{hp} + 10$ $ex_{mp} := xh_{mp} + 10$ $ex_{lp} := xh_{lp} + 1($

First stage cycle calculation.

$P := 4000$ $T := 1200$

$In_{hp} := \text{ST\_ptdata}(P, T, 1, 1)$ $cy_{hp} := \text{Rankine\_p}\left(In_{hp}, ex_{hp}, xh_{hp}, cl, P\right)$

$$cy_{hp} = \begin{pmatrix} 1 & 4000 & 4000 & 1144 & 1134 & 4000 \\ 0.05 & 1200 & 1200.301 & 786.655 & 785.83 & 1202.399 \\ 0.348 & 0.221 & 0.221 & 0.583 & 0.588 & 0.222 \\ 0.083 & 1388.585 & 1388.746 & 1250.646 & 1250.634 & 1389.869 \\ 0 & 1552.205 & 1552.412 & 1374.098 & 1374.098 & 1553.857 \\ 0.125 & 1.542 & 1.542 & 1.542 & 1.543 & 1.543 \\ 0.875 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\text{CycleWork}\left(cy_{hp}\right) = 179.3$

$\text{CycleEff}\left(cy_{hp}\right) = 12.8\,\%$

$$\frac{cy_{hp_{\boldsymbol{v},\boldsymbol{ex}}}}{cy_{hp_{\boldsymbol{v},\boldsymbol{cu}}}} = 2.6374$$

Second stage cycle calculation

$In_{mp} := \text{ST\_ptdata}\left(cy_{hp_{\boldsymbol{p},\boldsymbol{xh}}}, cy_{hp_{\boldsymbol{h},\boldsymbol{xh}}}, \boldsymbol{h}, 1\right)$ $cy_{mp} := \text{Rankine\_p}\left(In_{mp}, ex_{mp}, xh_{mp}, cl, In_{mp_{\boldsymbol{p}}}\right)$

$$cy_{mp} = \begin{pmatrix} 1 & 1134 & 1134 & 207 & 197 & 1134 \\ 0.05 & 785.83 & 787.949 & 386.232 & 383.107 & 797.855 \\ 0.229 & 0.588 & 0.59 & 2.219 & 2.331 & 0.597 \\ 0.145 & 1250.634 & 1251.667 & 1114.825 & 1114.825 & 1256.466 \\ 0 & 1374.098 & 1375.441 & 1199.814 & 1199.814 & 1381.684 \\ 0.177 & 1.543 & 1.544 & 1.544 & 1.549 & 1.549 \\ 0.823 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\text{CycleWork}\left(cy_{mp}\right) = 179.3$

$\text{CycleEff}\left(cy_{mp}\right) = 14.6\,\%$

$$\frac{cy_{mp_{\boldsymbol{v},\boldsymbol{ex}}}}{cy_{mp_{\boldsymbol{v},\boldsymbol{cu}}}} = 3.7616$$

Third stage cycle calculation

$In_{lp} := \text{ST\_ptdata}\left(cy_{mp_{\boldsymbol{p},\boldsymbol{xh}}}, cy_{mp_{\boldsymbol{h},\boldsymbol{xh}}}, \boldsymbol{h}, 1\right)$ $cy_{lp} := \text{Rankine\_p}\left(In_{lp}, ex_{lp}, xh_{lp}, cl, In_{lp_{\boldsymbol{p}}}\right)$

$$cy_{lp} = \begin{pmatrix} 1 & 197 & 197 & 24.696 & 14.696 & 197 \\ 0.05 & 383.107 & 407.893 & 239.395 & 212 & 485.447 \\ 0.124 & 2.331 & 2.43 & 14.702 & 24.219 & 2.717 \\ 0.396 & 1114.825 & 1127.051 & 989.827 & 991.149 & 1162.215 \\ 0 & 1199.814 & 1215.648 & 1057.013 & 1057.013 & 1261.271 \\ 0.258 & 1.549 & 1.567 & 1.567 & 1.618 & 1.618 \\ 0.742 & 1 & 1 & 0.891 & 0.904 & 1 \end{pmatrix}$$

$\text{CycleWork}\left(cy_{lp}\right) = 179.4$

$\text{CycleEff}\left(cy_{lp}\right) = 17.1\,\%$

$$\frac{cy_{lp_{\boldsymbol{v},\boldsymbol{ex}}}}{cy_{lp_{\boldsymbol{v},\boldsymbol{cu}}}} = 6.0493$$

The single stage engines expansion ratio:

$$\frac{cy_{v,ex}}{cy_{v,cu}} = 7.7118$$

Trying to get the engines expansion equal. Hard to do. Close enough. Could make MathCad do the balancing but would take time to develope the programming. The efficiencies are close enough. So going with this.

The compound combined stages expansion ratio:

$$\frac{cyhp_{v,ex}}{cyhp_{v,cu}} \cdot \frac{cymp_{v,ex}}{cymp_{v,cu}} \cdot \frac{cylp_{v,ex}}{cylp_{v,cu}} = 60.0134$$ Expansion doing work.

Compound total over all expansion ratio HP(first stage) cutoff point speciic volume to LP (last stage) end of expansion specific volume.

$$\frac{cylp_{v,ex}}{cyhp_{v,cu}} = 66.4898$$ Includes non work and work expansion.

The following figures second and third stage scaling parameters so as to use an equal amount steam.

$$S_{mp} := \frac{cyhp_{im,PA}}{cymp_{im,PA}} \qquad S_{mp} = 1.0628 \qquad S_{lp} := \frac{cyhp_{im,PA}}{cylp_{im,PA}} \qquad S_{lp} = 1.1782$$

The following is used to balance stage output.

$$\frac{CycleWO(cy_{hp}) + S_{mp} \cdot CycleWO(cy_{mp}) + S_{lp} \cdot CycleWO(cy_{lp})}{3} = 156.8734$$

Referance used for stage output. Each stages scaled output shouled equal this

Scaled output work of each stage in BTU:

Varied inter stage pressure to get these close.

$$CycleWO(cy_{hp}) = 156.862 \qquad S_{mp} \cdot CycleWO(cy_{mp}) = 156.8023 \qquad S_{lp} \cdot CycleWO(cy_{lp}) = 156.9561$$

Some values of interest

$$CycleWork(cy_{hp}) + S_{mp} \cdot CycleWork(cy_{mp}) + S_{lp} \cdot CycleWork(cy_{lp}) = 581.3102$$ BTU/lb of steam

$$\frac{CycleWO(cy_{hp}) + S_{mp} \cdot CycleWO(cy_{mp}) + S_{lp} \cdot CycleWO(cy_{lp})}{\left(cyhp_{h,in} - ST\_ptdata(14.696, 180, 1, 1)_{h}\right) cyhp_{im,PA}} = 38.3184\%$$ Engine efficiency

Down to figuring relative engine size for equal amounts of output work.

To do that I figure the output work in ftlb per cubic in of diplacement.

$$Work_{1S} := \frac{CycleWO(cy)\cdot \boldsymbol{J}}{cy_{\boldsymbol{v},\boldsymbol{ex}}\cdot 12\cdot 12\cdot 12}$$

The output work for the single stage engine per cubic inch.

$$Work_{3S} := \frac{\left(CycleWO\left(cy_{hp}\right) + S_{mp}\cdot CycleWO\left(cy_{mp}\right) + S_{lp}\cdot CycleWO\left(cy_{lp}\right)\right)\cdot \boldsymbol{J}}{\left(cy_{hp_{\boldsymbol{v},\boldsymbol{ex}}} + S_{mp}\cdot cy_{mp_{\boldsymbol{v},\boldsymbol{ex}}} + S_{lp}\cdot cy_{lp_{\boldsymbol{v},\boldsymbol{ex}}}\right)\cdot 12\cdot 12\cdot 12}$$

Same for three stage engne.

CycleWO figures the output work of the cycle for each pound of steam in the cycle. Here the pound of steam in the cycle includes both inlet and residual parts. The cycle function figure a cycle based in a pound of steam in the engine cycle. The total displacement is figured as stage displacements containing a pound of steam with the second and last stage scaled to use the same amount of inlet as the first stage. The displacement is based on the specific volume at the end of expansion point.

The output work is converted into foot pounds.

And the results are here:

$Work_{1S} = 29.8995$ ft lb/In^3 $Work_{3S} = 10.4591$ ft lb/In^3

Comparing the work output per cubic inch of displacement we find the compound produces 3.67 times the work of a single stage for a given displacement.

$$\frac{Work_{1S}}{Work_{3S}} = 2.8587$$

Or for equal output the 3 stage compound must have 2.86 times the displacement of the single stage.

$$Work_{1S}\cdot \frac{3600}{60\cdot 550} = 3.2618$$

HP/in^ @3600RPM

$$Work_{3S}\cdot \frac{3600}{60\cdot 550} = 1.141$$

HP/in^ @3600RPM

A look at the efficiency ratio.

$$\frac{\dfrac{CycleWO\left(cy_{hp}\right)+S_{mp}\cdot CycleWO\left(cy_{mp}\right)+S_{lp}\cdot CycleWO\left(cy_{lp}\right)}{\left(cy_{hp_{\boldsymbol{h},\boldsymbol{in}}} - ST\_ptdata(14.696, 180, 1, 1)_{\boldsymbol{h}}\right) cy_{lp_{\boldsymbol{im},\boldsymbol{PA}}}}}{CycleEff(cy)} = 1.9016$$